Comments on the Meta-Stable Vacuum in $N_f = N_c$ SQCD and Direct Mediation

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Abstract

We revisit $N_f = N_c$ SQCD and its non-supersymmetric minima conjectured by Intriligator, Seiberg and Shih (ISS). We argue that the existence of such minima depends on the signs of three non-calculable parameters and that no evidence can be inferred by deforming the theory. We demonstrate this by studying a deformation of the theory which involves additional gauge singlets. In this case, the conjectured minimum is destabilized. We comment on the consequences of such singlets in models of direct mediation and in particular in the Pentagon model.

1 Introduction

The idea of dynamical supersymmetry breaking (DSB) [1] provides an elegant explanation of the hierarchy problem. The earliest examples were given in [2, 3] and many more examples have been constructed since (for a review see e.g. [4, 5]). Still it has long been understood that models which exhibit stable DSB vacua are non-generic. Furthermore, while various tools for constructing DSB models exist [6] there is no systematic classification of such theories.

DSB models are the starting point for generating supersymmetry-breaking masses for the Standard Model (SM) superpartners. The breaking is communicated to the visible sector by either gravity, or gauge interactions (for a review see e.g. [7, 8]) and therefore only soft breaking is felt in the visible sector. Most scenarios suffer from various problems. For example, in gravity-mediated models, flavor changing contributions are not suppressed. Minimal anomaly-mediation [9, 10] leads to tachyonic sleptons, and while this problem can be solved, the solutions are typically fairly complicated.

Gauge mediated models, on the other hand, lead to viable soft masses, with no "hidden" assumptions [6, 11, 12]. Still, they are often deemed unattractive, since they involve several tiers of messenger fields, including gauge singlets, to mediate the breaking from the the DSB model to the standard model. Furthermore, the presence of the singlets often leads to new supersymmetric color-breaking minima (although the desired minimum is usually cosmologically stable) [13, 14]. These aesthetic shortcomings have led people to seek models of "direct gauge mediation", in which the standard model gauge group is embedded in the unbroken global symmetry of the DSB model [15, 16, 17, 18, 19, 20]. While this avenue is indeed more compact, and does not generate new unwanted minima, it typically results in Landau poles below the Planck scale. The reason is that, when the DSB model has a large enough unbroken global symmetry to accommodate the standard model, one finds too many new fields charged under the standard model gauge group.

Recently, these ideas regained a lot of attention [21, 22, 23, 24, 25, 26, 27, 28], following the elegant work of Intriligator, Seiberg and Shih (ISS) [29]. As ISS show, by abandoning the requirement of global supersymmetry breaking and allowing for meta-stable DSB vacua, one finds many more simple and generic calculable models. In particular, ISS study supersymmetric QCD (SQCD) with N colors and $N < N_f < 3N/2$ flavors, demonstrating that a meta-stable DSB vacuum is present near the origin of field space. The analysis strongly relies on weak-strong Seiberg duality [30] which provides a weakly coupled description of the theory.

These constructions open new avenues for model building. In particular, near the origin of field-space the global symmetry is large enough to allow for embedding the SM and therefore for new models of direct mediation. Nevertheless, such models still suffer from Landau poles unless the supersymmetry breaking scale is pushed to sufficiently large scales [26, 27, 29]. Such models therefore turn out to be rather complicated.

ISS also consider the particularly interesting case of SQCD with N colors and $N_f = N$ flavors. At low energy the theory is described by a non-linear sigma model with a quantum-deformed moduli space. Since these quantum corrections do not allow all fields to be close to the origin of moduli space, calculability is lost. Thus there is no weakly coupled description of the model that allows for establishing a meta-stable DSB vacuum. Deforming the theory (to $N_f = N + 1$) by adding another flavor restores control, and by doing so ISS conjecture that a meta-stable minimum exists also in the $N_f = N$ case.

The significance of this model lies in its minimal flavor symmetry. Gauging this symmetry potentially does not introduce Landau poles at low energy. Thus this model is interesting for phenomenological purposes. Indeed, soon after the ISS discovery, the Pentagon model [22, 31] was re-introduced, demonstrating a simple and attractive realization of direct mediation. Aside from the usual gauge dynamics, the model contains a singlet which plays an important role in obtaining a viable messenger spectrum, and which generates the μ-term as well.

The importance of such models calls for further study of the meta-stable minima in the $N_f = N$ case. In this paper we reexamine this case and its deformations. As we discuss at some length, the existence of a meta-stable minimum depends on the signs of three non-calculable coefficients which appear in the Kähler potential. In the deformation that ISS consider, these parameters are irrelevant by construction, so that the non-SUSY minimum is calculable. We suggest another deformation which is closely related to the Intriligator-Thomas-Izawa-Yanagida (ITIY) DSB model [32, 33]. As in the $N_f = N + 1$ deformation, there exists a region of parameter space where the theory is calculable and the ISS-like extremum (which coincides with the conjectured minimum as we approach $N_f = N$ SQCD) is found to be a saddle point rather than a minimum. This demonstrates the weakness of the conjecture, implying that the $N_f = N$ extremum is just as likely to be a saddle-point.

One approach towards settling this issue is to take advantage of the AdS/CFT correspondence. In [34] the $N_f = N$ model was realized on fractional branes placed on a Z_2 orbifold of the conifold. A gravity dual was suggested and found to posses a non-supersymmetric state,

indicating that the conjectured meta-stable minimum indeed exists. While clearly a step in the right direction, a complete gravity solution is still missing and more importantly, it is not clear whether such non-supersymmetric states remain in the transition between large 't Hooft and weak gauge coupling. We believe further work is needed in this regard.

The appearance of a saddle-point is directly related to the introduction of new gauge-singlet degrees of freedom. Large couplings to the singlets may destabilize the desired minimum. It is therefore natural to question the validity of direct-mediation models which take advantage of the $N_f = N$ scenario, and in particular of the Pentagon model [31, 22]. Unlike the deformation discussed above, this model is non-calculable so one cannot reliably establish the existence of a non-susy minimum with a viable messenger spectrum. Still, we argue that for small quark masses, such a minimum requires a large meson-singlet coupling, which would probably destabilize the minimum. Moreover, because of the large coupling, even if a minimum exists, it is not directly related to the ISS conjectured minimum.

The paper is organized as follows. In section 2 we review the ISS $N_f = N$ conjecture, emphasizing the $N_f = N + 1$ deformation and its relation to the original theory. In section 3 we consider a different deformation, in which the mesons and baryons are coupled to singlet fields. We show that the ISS-like extremum is in fact a saddle-point in the region of parameter space where the model is calculable. In Section 4 we consider direct mediation in the Pentagon model. Some details of the calculation are given in the Appendix.

2   The ISS conjecture

We begin this section with a quick review of the ISS supersymmetry-breaking minima for $SU(N)$ SQCD with $N + 1 < N_f \leq 3N/2$ [29]. For this range of N_f, the IR theory can be described by the weakly coupled "magnetic" theory, with $N_f - N$ colors and with the superpotential

$$W = \mathrm{Tr}m_Q M + \frac{1}{\widehat{\Lambda}}\mathrm{Tr}qM\bar{q} \ . \tag{1}$$

Here M corresponds to the meson of the original, "electric" theory, q, $\bar{q}$, are the magnetic quarks, and m_Q is the (electric) quark mass. The scale $\widehat{\Lambda}$ is related to the strong coupling scales of the electric and magnetic theories. The superpotential also contains non-renormalizable terms generated by non-perturbative effects. These are essential for seeing the supersymmetric

minima, but are negligible close to the origin. The potential is minimized at

$$M = 0, \quad q = -\bar{q} = \begin{pmatrix} q_0 \\ 0 \end{pmatrix}, \qquad q_0^2 = m_Q \,\widehat{\Lambda}\, \mathbf{1}_{N_f - N}, \qquad (2)$$

where M is an $N_f \times N_f$ matrix and q, $\bar{q}$ are $(N_f - N) \times N_f$. At the minimum, the F-terms for some M's are nonzero and supersymmetry is broken. Some of the dual quarks and mesons get mass at tree-level, through the cubic term of the superpotential (1). This cubic interaction also generates masses at the loop level for the remaining massless scalars (apart from the Goldstone bosons). For small m_Q, these masses are parametrically larger than contributions from non-calculable corrections to the Kähler potential. Thus, the theory near the origin is calculable by virtue of two important ingredients: (i) the smallness of m_Q, and (ii) the cubic superpotential interaction, which generates positive masses for all the scalar fields. This second ingredient is missing for $N_f = N$.

Let us consider then $N_f = N$. At low energy, the theory is described by a non-linear sigma model with the superpotential [35]

$$W = m_Q \text{Tr} M + \mathcal{A}(\det M - B\bar{B} - \Lambda^{2N}). \qquad (3)$$

Here for simplicity, we take the quark masses to be $m_{Q_{ij}} = m_Q \delta_{ij}$. The non-dynamical auxiliary field $\mathcal{A}$ is introduced to enforce the quantum constraint. The theory has N supersymmetric minima at

$$M_{ij} = \Lambda^2 \frac{(\det m_Q)^{1/N}}{m_Q} \, \delta_{ij}, \qquad B = \bar{B} = 0. \qquad (4)$$

As a first attempt at finding a non-supersymmetric minimum we extremize the potential on the baryonic branch, assuming a canonical Kähler potential. One finds a classical moduli space of solutions with $\mathcal{A} = 0$ and non-vanishing baryon number. We concentrate on the point with the largest flavor symmetry,

$$M = 0, \qquad B = -\bar{B} = \Lambda^N. \qquad (5)$$

The discussion below can be carried over to any other extrema. Around (5), only $B_- \equiv (B - \bar{B})/\sqrt{2}$ is massive due to the quantum constraint. The combination $B_+ \equiv (B + \bar{B})/\sqrt{2}$ [1], as well as all the mesons, remain massless: there is no superpotential coupling that can generate

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[1]Strictly speaking, the dynamical field is not B_+ but rather b where $B = \Lambda^N e^b$, $B = -\Lambda^N e^{-b}$ [29]. Still we will find it more convenient to work with B_+. The two parametrizations coincide to quadratic order.

masses for these fields. In order to discover the nature of this extremum, one must therefore take into account corrections to the Kähler potential. The only non-zero F-term is the meson F-term, F_M, so the relevant quantity is the $M - M^\dagger$ entry of the inverse Kähler metric. The Kähler potential is of the form

$$K = \frac{\text{Tr}M^\dagger M}{\Lambda^2} + \frac{(B_+ + B_+^\dagger)^2}{\Lambda^{2N-2}} + c_1 \frac{\text{Tr}M^\dagger M M^\dagger M}{\Lambda^4} + c_2 \frac{(\text{Tr}M^\dagger M)^2}{\Lambda^4} + c_3 \frac{(B_+ + B_+^\dagger)^2 \text{Tr}M^\dagger M}{\Lambda^{2N+2}} + \cdots . \quad (6)$$

Here c_1, c_2 and c_3 are order-one parameters[2], and the ellipses stand for terms which are irrelevant for our discussion. The form of (6) follows from the flavor, baryon and non-anomalous $\mathbb{Z}_{2N}$ axial symmetries. The last three terms in (6) are small for $M/\Lambda^2, B_+/\Lambda^N \ll 1$. However, they are the only source of meson and B_+ masses. Indeed, the potential takes the form,

$$V \sim \left(1 + \alpha \frac{\text{Tr}M^\dagger M}{\Lambda^4} + \tilde\beta \frac{\text{Tr}M\text{Tr}M^\dagger}{\Lambda^4} + \gamma \frac{(B_+ + B_+^\dagger)^2}{\Lambda^{2N}} \right) |m_Q \Lambda|^2 + ..., \quad (7)$$

where the coefficients α, $\tilde\beta$, and γ depend on c_1, c_2 and c_3. Therefore the above corrections contribute order $|m_Q|^2 \ll \Lambda^2$ to the masses-squared of the canonically-normalized meson and baryon. In order for this extremum to be a minimum, we must have

$$\alpha > 0 , \quad \beta \equiv \alpha/N + \tilde\beta > 0 , \quad \gamma > 0 . \quad (8)$$

However, as ISS discuss, the theory is strongly coupled at the scale Λ, and so α, β and γ are non-calculable. At the field theory level one can therefore only conjecture the existence of a supersymmetry-breaking minimum near the origin. Moreover, studying other minima far away from the origin at $M/\Lambda \sim 1$ requires the knowledge of higher order terms in the Kähler potential.

To make further progress, ISS deformed the theory by adding another flavor. This is the $N_f = N + 1$ case which we now discuss. To understand this deformation, one must carefully follow the corrections to the Kähler potential, as the mass of the extra flavor is dialed. To be concrete, let us take $m_{Q_{ij}} = \text{diag}(m_Q, .., m_Q, m_{N+1})$ with $m_{N+1} \geq m_Q$. At low energy, this theory too is described by a non-linear sigma model in terms of the baryons and mesons [35]. The superpotential is identical to the superpotential of the $N_f > N + 1$ magnetic theory with the dual quarks replaced by the baryons [35],

$$W = \frac{1}{\widehat\Lambda^{2N-1}} \left(\widehat B \widehat M \widehat{\widehat B} - \det \widehat M \right) + \text{Tr} m_Q \widehat M. \quad (9)$$

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[2] For simplicity, we ignore order-one coefficients in front of the leading terms in the Kähler potential. Thus one should not confuse the above parametrization with the one of [29].

Here $\widehat{B}$ and $\widehat{\bar{B}}$ are the $N+1$ baryons, $\widehat{M}$ are the mesons of the deformed theory and $\widehat{\Lambda}$ is the scale at which the theory becomes strongly coupled. Unlike in the discussion of the magnetic theory at the beginning of this section, here we chose to display the non-renormalizable term $\det \widehat{M}$ since it is important for recovering the $N_f = N$ superpotential. Still, as before, this term will play no role in the analysis near the origin.

To make contact with the $N_f = N$ theory, it is convenient to write the $N_f = N+1$ (hatted) fields as

$$\widehat{M} = \begin{pmatrix} M_i^j & \widehat{M}_i^{N+1} \\ \widehat{M}_{N+1}^j & M_{N+1}^{N+1} \end{pmatrix}, \tag{10}$$

$$\widehat{B} = (B^i, B) \qquad \widehat{\bar{B}} = (\bar{B}^i, \bar{B}) . \tag{11}$$

As $m_{N+1} \to \infty$, the heavy flavor can be integrated out, leaving only M, B and $\bar{B}$ light. In this limit, the theory reduces to the original $N_f = N$ case with the identification $\mathcal{A} = M_{N+1}^{N+1}/\widehat{\Lambda}^{2N-1}$ and $\Lambda^{2N} = m_{N+1}\widehat{\Lambda}^{2N-1}$. Indeed, for constant Λ, the limit $m_{N+1} \to \infty$ corresponds to $\widehat{\Lambda} \to 0$ which sets the kinetic term of $\mathcal{A}$ to zero making it non-dynamical.

For finite m_{N+1} however, M_{N+1}^{N+1} must be treated as a dynamical field. As before, one may try to minimize the tree-level potential first, ignoring corrections to the Kähler potential. The analysis is identical to the the analysis of the $N_f > N+1$ theory, only now one quark mass is different. Again, we concentrate on the extremum,

$$\widehat{M} = 0, \qquad B^i = \bar{B}_i = 0, \qquad B = -\bar{B} = \Lambda^N, \tag{12}$$

just as for $N_f = N$. But as opposed to the $N_f = N$ theory, the superpotential (9) contains a cubic term. At tree level, this term generates a mass-squared of order $m_{N+1}\widehat{\Lambda}$ for all fields apart from M and $B_+ \equiv (B+\bar{B})/\sqrt{2}$. The latter, just as for the case of more flavors, become massive at the one loop level, with masses of order $m_Q^2\widehat{\Lambda}/m_{N+1}$. To see this, note that the only fields with non-zero F-terms are M_i^i ($i \leq N$), with $F \sim m_Q\widehat{\Lambda}$. As a result, the fields B_i and $\bar{B}^i$ have supersymmetric masses-squared of order $m_{N+1}\widehat{\Lambda}$, and supersymmetry-breaking masses-squared of order $m_Q\widehat{\Lambda}$. These fields then generate a non-zero supertrace, leading to masses for M and B_+,

$$m_{\text{loop}}^2 \sim \frac{1}{16\pi^2} \frac{m_Q^2\widehat{\Lambda}}{m_{N+1}} . \tag{13}$$

This is the crucial difference between the original $N_f = N$ model and the deformation: in the deformed theory, just as for larger values of N_f, all scalars apart from the Goldstones get

masses either at tree-level or at one-loop, and the pseudo-flat directions are (at least naively) lifted, giving a minimum at (12).

However, on top of these mass terms, one must still consider the corrections to the Kähler potential. As in eqn. (7), these contribute $\delta m^2 \sim m_Q^2$ and are therefore negligible compared with (13) *as long as* $m_{N+1} \ll \widehat{\Lambda}$. Thus for sufficiently small m_{N+1}, we can reliably establish a true minimum. On the other hand, for $m_{N+1} \geq \widehat{\Lambda}$, the signs of the coefficients α, β and γ of eqns. (7), (8) (with Λ replaced by $\widehat{\Lambda}$) are crucial.

Let us therefore summarize the essence of the ISS conjecture. Whether the point (5) is a minimum or not depends on the signs of unknown parameters, α, β and γ. One can deform the theory by adding tree-level couplings which stabilize the above extremum by generating positive masses-squared for all fields. The deformation can be worked out in a limit where the above parameters are not important and can be neglected.

Clearly, the deformation gives us no information on α, β and γ. It is therefore just as likely that one or more of the mesons and baryons is tachyonic. Physically this would amount to a smooth transition in the potential as the minimum becomes a saddle-point when m_{N+1} crosses $\widehat{\Lambda}$ from below. To emphasize this point, we now consider a different deformation of the $N_f = N$ theory, with the mesons and baryons coupled to singlet fields. As we will see, when the deformed theory is calculable, the extremum (5) turns out to be a saddle point, demonstrating that such a transition indeed occurs. We therefore conclude that no information can be extracted on the nature of the $N_f = N$ supersymmetry-breaking extremum by deforming the theory.

3 Adding singlets

We now deform the ISS model by adding singlet fields S_{ij}, T and $\bar{T}$ with superpotential couplings to the mesons and baryons,

$$W = m_Q \text{Tr} M + \lambda \text{Tr} SM + \kappa (TB + \bar{T}\bar{B}) + \frac{1}{2} m_S \text{Tr} S^2 + \frac{1}{2} m_T (T^2 + \bar{T}^2) \,. \tag{14}$$

This is nothing but the Intriligator-Thomas-Izawa-Yanagida model (ITIY) [32, 33], with singlet mass terms added. Without these mass terms, the quark masses can be absorbed by a shift redefinition of the singlets S_{ij}.

As we will see below, the model has a local non-supersymmetric extremum similar to the minimum conjectured by ISS. As the singlets decouple, the model approaches $N_f = N$ SQCD, and the local supersymmetry-breaking extremum approaches the ISS-conjectured minimum (5).

We can decouple the singlets either by decreasing their superpotential couplings to the mesons and baryons, or by increasing their masses,

$$\lambda \to 0 \quad \text{or} \quad m_S \to \infty \, ; \quad \kappa \to 0 \quad \text{or} \quad m_T \to \infty \, . \tag{15}$$

However, as we will see below, there is a lower bound on the couplings λ, κ, and equivalently, an upper bound on the masses m_T, m_S. For very small couplings (or very large masses), non-calculable Kähler corrections become important and we cannot reliably study the ISS-like extremum, much like in the $N_f = N + 1$ deformation. Still, as long as the model is calculable, we will find that this extremum is a saddle point rather than a minimum.

3.1 Supersymmetric minima

Before going on, it is useful to recall what happens in the ITIY model. The classical super-potential of the model is given by (14) with m_S and m_T set to zero. Supersymmetry is then broken, since the singlet F-terms only vanish when the mesons and baryons are at the origin, in conflict with the quantum-modified constraint. Defining again $T_\pm = (T \pm \bar{T})/\sqrt{2}$, one finds that at tree-level, T_- is a flat direction. This degeneracy is lifted at the loop-level, and as argued in [36], the loop corrections can be reliably computed near the origin. Indeed these loop corrections are generated by light states, and scale as $\mathcal{O}(\kappa^4)$, while non-calculable corrections from states at the scale Λ are suppressed by $\mathcal{O}(\kappa^6)$ [36]. Thus for sufficiently small κ, all fields are stabilized at the origin, apart from $B_- = \sqrt{2}\Lambda^N$. Since the only nonzero F-term is F_{T_-}, the Goldstino is the T_- fermion.

As discussed above, here we add singlet mass terms. As these masses are turned on, supersymmetric vacua move in from infinity, and the theory can only have *local* supersymmetry-breaking minima at best. Taking into account the quantum-modified constraint,

$$W_{NP} = \mathcal{A} \left(\det M - B\bar{B} - \Lambda^{2N} \right) , \tag{16}$$

one finds three families of supersymmetric solutions. The first is given by

$$B_\pm = T_\pm = 0 \, , \qquad |M| = \Lambda^2 \, , \qquad |S| = -\frac{\lambda}{m_S} \Lambda^2 \, . \tag{17}$$

The second is given by, up to terms of order λ^2,

$$B_+ = T_+ = 0 \, , \qquad M \simeq - \left(\frac{m_Q m_T}{\kappa^2} \right)^{1/(N-1)} , \qquad S \simeq \frac{\lambda}{m_S} \left(\frac{m_Q m_T}{\kappa^2} \right)^{1/(N-1)} ,$$

$$B_-^2 = -\det M + \Lambda^{2N} \, , \qquad T_- = -\frac{\kappa}{m_T} B_- \, , \tag{18}$$

8

and the third solution is obtained from the second for $B_+^2 \leftrightarrow -B_-^2$, $T_+ \leftrightarrow T_-$, and $M \to -M$, $S \to -S$. Clearly, in the decoupling limit (15), only the first solution remains at a finite distance from the origin. The other two solutions approach the classical solutions with the meson VEVs running to infinity.

3.2 Non-supersymmetric saddle points

We are now ready to look for the ISS conjectured minimum. For now, we will assume that the Kähler potential is canonical in all fields. We will later examine the region of validity of this approximation. Strictly speaking, around a given solution we should use the constraint to eliminate the heavy degree of freedom, say B_-, and derive the potential for the remaining degrees of freedom. In the process, various non-renormalizable interactions of the remaining fields will be induced, making the potential quite unwieldy. We will therefore first perform the analysis with the Lagrange multiplier in place, and later explain how the results are modified in the full analysis (this analysis is described in detail in the Appendix).

It is simple to verify that at the minimum, the F-terms of $B_\pm$, $T_\pm$ and $\mathcal{A}$ all vanish, resulting in two possible solutions at[3],

$$B_\pm = T_\pm = 0 \,, \qquad \mathcal{A} = \pm \frac{\kappa^2}{m_T} \,, \qquad B_\mp^2 = \mp 2(\det M - \Lambda^{2N}) \,, \qquad T_\mp = -\frac{\kappa}{m_T} B_\mp \,. \qquad (19)$$

Since we are interested in extrema which preserve the $SU(N)_{\text{diag}}$ global symmetry, we take the ansatz,

$$M_{ij} = M \,\delta_{ij} \,, \qquad S_{ij} = S \,\delta_{ij} \,. \qquad (20)$$

One therefore obtains an effective potential for M and S,

$$V = N \left| \lambda S + m_Q \pm \frac{\kappa^2}{m_T} M^{N-1} \right|^2 + N \left| \lambda M + m S \right|^2 \,. \qquad (21)$$

with a non-supersymmetric extremum at,

$$M = \left(\pm \frac{\lambda^2}{(N-1)m_S} \frac{m_T}{\kappa^2} \right)^{\frac{1}{N-2}} \,, \qquad (22)$$

$$S = -\frac{\lambda^*}{|\lambda \Lambda|^2 + |m_S|^2} \left[m_Q \Lambda^2 + \left(\pm \frac{\lambda^2}{(N-1)m_S} \frac{m_T}{\kappa^2} \right)^{\frac{1}{N-2}} \left(\frac{\lambda}{\lambda^*} m_S^* + \frac{\lambda^2 \Lambda^2}{(N-1)m_S} \right) \right] \,.$$

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[3]There is another uninteresting solution with $B_+ = B_- = T_+ = T_- = 0$. This solution is on the mesonic branch and is not related to the ISS conjecture.

We now wish to relate the above solutions to the ISS extremum. We therefore consider the case $\mathcal{A} > 0$. To this end, one may take the decoupling limit, (15), in various ways thereby probing the space of vacua in the original $N_f = N$ SQCD. To discover the nature of the extrema, one then needs to compute the mass spectrum for each given decoupling limit. In particular, to approach the ISS extremum, the singlets S should decouple faster than $T_\pm$, for example by taking

$$\lambda, \kappa \to 0 \,, \qquad \frac{\lambda}{\kappa} \to 0 \,. \tag{23}$$

The only nonzero F-terms at this extremum are F_M and F_S, with $F_M \sim m_Q$ for small λ. Thus the Goldstino is a mixture of the M and S fermions, and it tends to the mesino when $\lambda \to 0$, as expected for the ISS minimum.

We can now analyze the nature of this extremum. To leading order in λ, the (S, M) mass-squared matrix takes the simple form,

$$m_{\text{bosons}}^2 = N \begin{pmatrix} |\lambda\Lambda|^2 & \lambda^*\Lambda m_S & \xi_{\text{SB}} & 0 \\ \lambda m_S^*\Lambda & |m_S|^2 & 0 & 0 \\ \xi_{\text{SB}}^* & 0 & |\lambda\Lambda|^2 & \lambda\Lambda m_S^* \\ 0 & 0 & \lambda^* m_S\Lambda & |m_S|^2 \end{pmatrix} \,, \tag{24}$$

where

$$\xi_{\text{SB}} \simeq (N-1)(N-2)\frac{\kappa^2}{m_T} \left(\frac{\lambda^2}{(N-1)m_S} \frac{m_T}{\kappa^2} \right)^{\frac{N-3}{N-2}} m_Q^* \Lambda^4 \,, \tag{25}$$

is the supersymmetry-breaking contribution. The determinant of the matrix above is negative, so there is at least one tachyonic direction. In fact it is easy to see that there is precisely one such direction. Note that the determinant of the diagonal 2×2 block, which coincides with the fermion mass matrix, is exactly zero, signalling the presence of the Goldstino. The S and M fermions mix to give one massive state, which is predominantly S of mass near m_S, and one massless fermion, the Goldstino, which is mostly M. The supersymmetry-breaking contribution ξ_{SB} results in splittings between the fermions and scalars. For small λ, this splitting occurs mostly in the M sector. Since however the supertrace still vanishes (we are working at tree-level) one scalar becomes lighter than the Goldstino, with a tachyonic mass $m^2 \sim -|\xi_{\text{SB}}|$.

As we saw in section 2, the meson masses also receive contributions from non-calculable Kähler terms, which are of order $|F_M|^2/\Lambda^2 \sim m_Q^2$ [see eqn. (7)][4]. For the model to be calculable,

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[4] In fact, due to the new interactions with the singlets, there are additional non-calculable contributions to the Kähler potential which are negligible for small λ, κ.
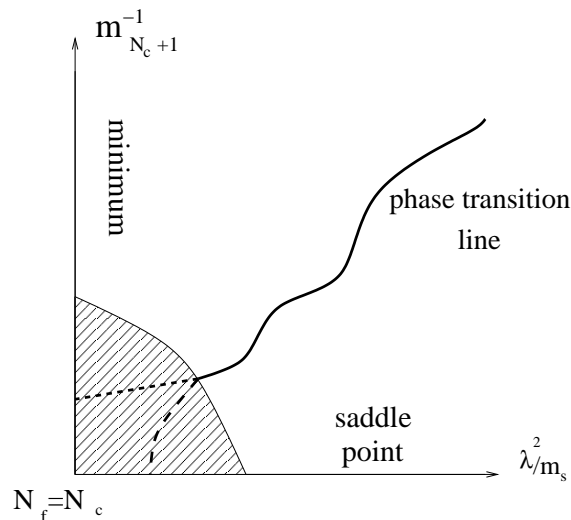
Figure 1: The two deformations of $N_f = N$ SQCD. The transition between a saddle point and a minimum occurs in the shaded region where calculability is lost.

these contributions must be smaller than the smallest eigenvalue of the mass-matrix (24),

$$\xi_{\mathrm{SB}} \gg m_Q^2 \ . \tag{26}$$

One can choose, for example (for large N),

$$m_Q \ll \lambda \Lambda \ll m_S \lesssim \Lambda \ . \tag{27}$$

A similar bound holds in the baryonic sector for κ^2/m_T. Thus we see that we cannot decouple the singlets completely while preserving the calculability of the model. There is a lower bound on the coupling λ, or, alternatively an upper bound on the mass m_S. Outside the allowed range, the signs of the parameters α, β and γ become crucial for establishing a minimum. In this regard the above deformation is on exactly the same footing as the deformation considered by ISS. This situation is depicted in figure 3.2.

As we noted above, so far we worked with the Lagrange multiplier for simplicity. In the Appendix we present a more careful analysis, where we eliminate one of the fields using the constraint from the start. Indeed, the location of the extremum (19),(22), and the mass-squared matrix (24) are corrected by small amounts, but the conclusion remains unchanged.

One could still hope that the instability we found might be cured by positive contributions arising from the Coleman-Weinberg (CW) potential. The situation is different from the $N_f \geq$

$N + 1$ case. As we reviewed in the previous section, for $N_f = N + 1$ there are pseudo-flat directions which are lifted by the dominant, calculable one-loop corrections. In our case on the other hand, there are no such flat directions at tree level. Furthermore, the one-loop contributions are smaller than the tree-level ones. Note that the only contribution to the CW potential is from the fields M and S, since the masses of the remaining fields are approximately supersymmetric. Thus the CW potential is roughly

$$\Delta V = \frac{1}{64\pi^2} \text{Str} M^4 \log \frac{M^2}{\Lambda^2} \propto \frac{1}{64\pi^2} \xi_{\text{SB}}^2 . \tag{28}$$

This correction is one loop-suppressed compared with the tree-level contribution to the tachyonic mass and thus cannot stabilize the extremum.

Finally, one could ask whether the CW potential can generate a distinct minimum which coincides with the ISS minimum in the decoupling limit. While a minimum is indeed generated for sufficiently small singlet mass m_S, in the decoupling limit this minimum is infinitely far in field space from the ISS and supersymmetric minima. The simplest way to see this, is to consider the one loop correction to the Kähler potential for S,

$$\delta K \sim -\frac{N}{32\pi^2} |\lambda S + m_Q|^2 \log \frac{|\lambda S + m_Q|^2}{\Lambda^2} . \tag{29}$$

Therefore for sufficiently small mass, m_S, a local minimum is generated at $S \sim -m_Q/\lambda$. In the decoupling limit this minimum is driven to infinity. Hence, it cannot correspond to the ISS conjectured minimum.

4   Direct mediation with singlets?

As discussed in the introduction, one of the main virtues of the ISS supersymmetry-breaking minima is that many fields are at the origin. There is thus a large unbroken global symmetry, which makes these theories promising starting points for models of direct gauge mediation [15, 16, 17, 18, 19, 20]. In particular, the most compact model which potentially does not lead to Landau poles for the standard model couplings at low energy is the $N_f = N$ case. Recently such a model has been proposed, taking advantage of the conjectured minimum in $N_f = N$ SQCD [22][5]. The model is based on the specific case of $N_f = N = 5$ (and hence dubbed the "Pentagon model"), with the SM gauge group embedded in the $SU(5)_{\text{diag}}$ global symmetry.

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[5]In fact, an earlier version of the model [31], which is not based on the ISS minimum, involves a meta-stable supersymmetry-breaking minimum in the context of "Cosmological Supersymmetry Breaking" [37].

One gauge singlet, S, is added to the model, in order to generate the μ-term through the superpotential coupling SH_uH_d. S obtains a VEV of order the supersymmetry-breaking F-terms, which are chosen to be $\mathcal{O}(100\,\text{GeV})$ thus solving the μ-problem. In fact, the use of singlets is common for solving this problem in models of direct mediation (see, e.g. [12]). As we will discuss, this singlet also plays an important role in generating a viable messenger spectrum.

In the previous section we showed that in the presence of large singlet couplings, the ISS extremum at $M = 0$ may be destabilized. It is therefore natural to ask whether the same is true for the Pentagon model. While the model is non-calculable, we will argue that for small quark masses, the singlet coupling must be sufficiently large in order to avoid negative contributions to the MSSM scalars. Therefore, destabilization is likely to occur.

To see first that non-calculable corrections are crucial in this setup, let us briefly review the model. The model has just one gauge singlet S. The relevant part of the superpotential is,

$$W = m_Q \text{Tr} M + \lambda S \text{Tr} Y M + \frac{1}{6} g S^3 + \mathcal{A} \left(\det M - \frac{B_+^2}{2} + \frac{B_-^2}{2} - \Lambda^{10} \right) \tag{30}$$

Here Y_{ij} is the hypercharge generator, normalized to be $Y_{ij} = \text{diag}(1, 1, 1, -3/2, -3/2)$. As before, it is straightforward to check that near the origin of the mesonic direction, the potential is extremized at $M_{ij} = 0$ for $i \neq j$. Furthermore, given the $(SU(3) \times SU(2))_{\text{diag}}$ symmetry, the ansatz we are seeking is of the form,

$$M_{ij} = M_d \delta_{ij} + M_Y Y_{ij}. \tag{31}$$

Ignoring first higher order terms in the Kähler potential, the potential is extremized along the baryonic branch at $S = M_Y = 0$, with $\mathcal{A} = 0$ and M_d undetermined.

Near the origin, higher order Kähler terms will generate supersymmetry-breaking masses-squared of order $F_{M_d}^2 \sim m_Q^2$ for the mesons, just as in eqn. (7). They will also shift $\mathcal{A}$ from zero, so that $\mathcal{A} \propto F_{M_d}$. This in turn will generate masses from the $\mathcal{A} \det M$ term in the superpotential, for both the fermion and scalar mesons. Clearly however, as long as the mesons are close to the origin, these tree-level contributions cannot dominate over the Kähler contributions. For *small* λ, the existence of a minimum therefore depends on the signs of the parameters α, β and γ.

Imagine then that λ is small, and that α, β and γ are such that a minimum is generated. As we discussed above, the SM gauge group is embedded in the $\text{SU}(5)_{diag}$ flavor symmetry. Below Λ, the messengers of gauge mediation are therefore the mesons. These get Dirac masses from two sources. The first is the $\mathcal{A} \det M$ term discussed above, and the second is higher-dimension Kähler terms, such as the third and fourth terms of eqn (6). Both contributions

are proportional to m_Q and some positive power of λ. In addition, the scalar mesons have supersymmetry-breaking masses of order m_Q. For small λ, the messenger supertrace is therefore positive. Furthermore, for $\lambda \ll 1$ and $m_Q \ll \Lambda$, there is some region of energies in which the messengers are weakly coupled. The positive supertrace then generates a negative contribution to the masses of MSSM scalars [38]. This contribution arises at one-loop, and is logarithmically enhanced as $\log(\Lambda_{UV}/m_F)$, where Λ_{UV} is the appropriate cutoff, and m_F is the messenger scale. In the case at hand, $\Lambda_{UV} \sim \Lambda$, where the positive supertrace is canceled by additional strongly-interacting fields charged under the SM gauge group. Of course close to Λ the theory becomes strongly interacting, and there will be non-calculable corrections to the soft masses. Nonetheless, for a large enough scale separation the negative contribution would win because of the logarithmic enhancement. We conclude that the coupling λ cannot be too small. For λ of order one, the minimum would probably be destabilized, much like we found in section 3. In any case, such a large coupling drives the mesons to VEVs of order Λ^2. Thus Kähler corrections are important to all orders, and the minimum required for the Pentagon no longer depends merely on α, β and γ. It is therefore not directly related to the ISS conjecture.

Finally, note that we assumed here $m_Q \ll \Lambda$. In this regime, the ISS analysis for $N_f > N$ is reliable, and the lifetime of the minimum is parametrically enhanced. In [22], m_Q is taken to be of order Λ, and the spectrum cannot be reliably computed.

5   Conclusions

ISS conjecture a DSB minimum for $N_f = N$ SQCD. They reach this conclusion by deforming the theory with an additional flavor. The importance of this conjecture lies in its appeal for model building and in particular for constructing models of direct mediation which do not suffer from Landau poles at low energy. In this paper we revisited this conjecture. We argued that deforming the theory gives, by construction, no information on the existence of such a minimum and therefore there is no evidence for a DSB vacuum. In particular, the existence of this state depends on the signs of three non-calculable parameters in the Kähler potential.

To demonstrate our point, we studied another deformation by coupling singlets to the mesons and baryons of the theory. For sufficiently large couplings the theory is calculable close to the origin. As we showed, the would-be ISS minimum is destabilized by the presence of the singlets and becomes a saddle point. Two conclusions are to be inferred from this deformation: (i) As we dial couplings, a minimum in one theory becomes a saddle point in another. This

transition occurs in a region where the theory is non-calculable. This is in accord with our claim that no information can be extracted on the existence of a minimum in the original theory. (ii) Coupling singlets to such gauge theories can quite generically destabilize existing minima.

Given the latter conclusion we briefly discussed direct mediation based on $N_f = N$ SQCD, assuming that a minimum does exist. An example of such a model is the Pentagon model presented in [22]. We argued that the coupling to the singlet cannot be too small in this case. On the other hand, a large coupling would drive the mesons far from the origin where both the tree-level and non-calculable corrections are important. Thus while likely, one cannot conclude whether similar destabilization occurs in this model. Still the existence of the minimum depends on the complete structure of the Kähler potential and is unrelated to the original ISS minimum at the origin.

6 Acknowledgments

A The ISS-like extremum: full analysis

In 3.2 we presented a somewhat simplified analysis of the $N_f = N$ theory coupled to singlets, keeping the Lagrange multiplier in the theory and treating it on equal footing with the other fields. Here we will refine this analysis, and impose the constraint right away to eliminate the heavy field B_-, whose mass is of order Λ. For convenience, we will set $\Lambda = 1$, such that all fields are dimensionless. The quantum modified constraint then gives

$$B_- = \sqrt{2 - 2M^N + B_+^2} \ . \tag{32}$$

Using the parametrization (20) the superpotential is then

$$W_{eff} = N\lambda MS + \kappa T_+ B_+ + \kappa T_- \sqrt{2 - 2M^N + B_+^2} + Nm_Q M + \frac{N}{2} m_S S^2 + \frac{m_T}{2} \left(T_+^2 + T_-^2\right), \quad (33)$$

The potential is extremized for

$$0 = \frac{\partial V}{\partial S} = Nm_S F_S^* + N\lambda F_M^* \qquad (34)$$

$$0 = \frac{\partial V}{\partial M} = N\lambda F_S^* + \frac{N\kappa T_- B_+ M^{N-1}}{B_-^3} F_{B_+}^* - \frac{N\kappa M^{N-1}}{B_-} F_{T_-}^* - \qquad (35)$$

$$\frac{N(N-1)\kappa T_- M^{N-2}}{B_-} F_M^* - \frac{N^2 \kappa T_- M^{2N-2}}{B_-^3} F_M^*$$

$$0 = \frac{\partial V}{\partial T_-} = m_T F_{T_-}^* - \frac{N\kappa M^{N-1}}{B_-} F_M^* + \frac{\kappa B_+}{B_-} F_{B_+}^* \qquad (36)$$

$$0 = \frac{\partial V}{\partial T_+} = m_T F_{T_+}^* + \kappa F_{B_+}^* \qquad (37)$$

$$0 = \frac{\partial V}{\partial B_+} = \frac{\kappa T_-}{B_-} F_{B_+}^* - \frac{\kappa T_- B_+^2}{B_-^3} F_{B_+}^* + \kappa F_{T_+}^* + \qquad (38)$$

$$\frac{\kappa B_+}{B_-} F_{T_-}^* + \frac{N\kappa T_- B_+ M^{N-1}}{B_-^3} F_M^*$$

where we use B_- to denote the combination (32) for convenience. The last two equations hold if we choose

$$F_{T_+} = F_{B+} = 0 , \qquad (39)$$

and therefore

$$B_+ = T_+ = 0 . \qquad (40)$$

Thus, B_+ and T_+ remain as in (19), and their F-terms still vanish. At tree level, there are therefore no mass terms that mix the (B_+, T_+) sector with the (M, S) sector, just as we found in section 3.2 However, a $T_- - M$ mixing is generated now.

Since it is difficult to solve the remaining equations exactly, we will study the theory in the decoupling limit, as an expansion for small λ. It is convenient to choose m_S and m_T of order one. In view of the discussion in section 3, we want λ to be smaller than κ. To maintain calculability we also choose $m_Q \sim \lambda^2$. We first note that F_T shifts from zero since otherwise eqn (36) isn't satisfied. To solve this equation we take the ansatz

$$T_- = -\frac{\kappa}{m_T} B_- + \frac{\delta T_-}{m_T^2} \qquad (41)$$

16

which gives,

$$m_T F_{T_-}^* = \delta T_-^* = \frac{N \kappa M^{N-1}}{B_-} F_M^*. \tag{42}$$

Furthermore, in the decoupling limit M is small (we will see below that it is of order $\lambda^{\frac{2}{N-2}}$), so we can safely neglect terms of order $\left(M^{2N}\right)$ [6]. Using the above, we find to leading order,

$$M \simeq \left(\frac{\lambda^2}{m_S} \frac{m_T}{\kappa^2} \frac{1}{(N-1)} \right)^{\frac{1}{N-2}} \tag{43}$$

just as in eqn (22), and

$$S = -\frac{\lambda^*}{|m_S|^2 + |\lambda|^2} \left(m_Q + \frac{\lambda}{\lambda^*} m_S^* M - \frac{\kappa T_- M^{N-1}}{B_-} \right) \simeq -\frac{\lambda^* m_Q}{|m_S|^2} - \frac{\lambda^* M}{m_S} \tag{44}$$

It is easy to see that this coincides with the solution (22).

Having found the extremum, we can now calculate the bosonic mass-squared matrices. As we mentioned above, at tree-level, there is no mixing between B_+, T_+ and the remaining fields. The (B_+, T_+) mass matrix is

$$m_{BT}^2 = \begin{pmatrix} |\kappa|^2 + \frac{|\kappa|^4}{|m_T|^2} & m_T \kappa^* - \frac{\kappa^{*2}\kappa}{m_T^*} & \xi_{BT} & 0 \\ m_T^* \kappa - \frac{\kappa^2 \kappa^*}{m_T} & |\kappa|^2 + |m_T|^2 & 0 & 0 \\ \xi_{BT}^* & 0 & |\kappa|^2 + \frac{|\kappa|^4}{|m_T|^2} & m_T^* \kappa - \frac{\kappa^2 \kappa^*}{m_T} \\ 0 & 0 & m_T \kappa^* - \frac{\kappa^{*2}\kappa}{m_T^*} & |\kappa|^2 + |m_T|^2 \end{pmatrix} \tag{45}$$

where

$$\xi_{BT} = \delta T_- \frac{N \kappa M^{N-1}}{m_T^2 B_-^3} F_M^* \ll \xi_{\mathrm{SB}} \tag{46}$$

As usual the diagonal blocks of this matrix are the mass matrices for the fermions. The off-diagonal terms involve the supersymmetry breaking F-term, and are parametrically small. In fact, they are smaller than the supersymmetry-breaking contributions in the M, S sector, ξ_{SB}. Note that this remains true when non-calculable contributions of the form (6) are taken into account. The latter induce susy-breaking masses for B_+ which are of order m_Q and therefore larger than ξ_{BT} but smaller than ξ_{SB}. Clearly, all eigenvalues of this matrix are positive and no instability develops here.

We now turn to the second sector which contains the fields M, S and T_-. Calculating the boson matrix, substituting (43,44) for the VEVs and neglecting terms of order M^{2N} or higher

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[6]For convenience we work here in a limit of large N.

we get

$$m_0^2 = N^2 \begin{pmatrix} |\lambda|^2 + \left|\frac{\lambda^2}{m_S}\right|^2 + |\Omega|^2 & m_S^*\lambda + \frac{\lambda^*\lambda^2}{m_S} & \Omega^*\frac{\lambda^2}{m_S} + \frac{m_T^*}{N}\Omega & \xi_{\mathrm{SB}} & 0 & m_{MT_-}^2 \\ m_S\lambda^* + \frac{\lambda^{*2}\lambda}{m_S^*} & |m|^2 + |\lambda|^2 & \Omega^*\lambda & 0 & 0 & 0 \\ \Omega\frac{\lambda^{*2}}{m_S^*} + \frac{m_T}{N}\Omega^* & \Omega\lambda^* & \frac{|m_T|^2}{N^2} + |\Omega|^2 & m_{MT_-}^2 & 0 & 0 \\ \xi_{\mathrm{SB}} & 0 & m_{MT_-}^{*2} & |\lambda|^2 + \left|\frac{\lambda^2}{m_S}\right|^2 + |\Omega|^2 & m_S\lambda^* + \frac{\lambda\lambda^{*2}}{m_S^*} & \Omega\frac{\lambda^{*2}}{m_S^*} + \frac{m_T}{N}\Omega^* \\ 0 & 0 & 0 & m_S^*\lambda + \frac{\lambda^2\lambda^*}{m_S^*} & |m|^2 + |\lambda|^2 & \Omega\lambda^* \\ m_{MT_-}^{*2} & 0 & 0 & \Omega^*\frac{\lambda^2}{m_S} + \frac{m_T^*}{N}\Omega & \Omega^*\lambda & \frac{|m_T|^2}{N^2} + |\Omega|^2 \end{pmatrix}$$

where ξ_{SB} is as in (25) and we defined

$$\Omega \equiv \frac{1}{N}\frac{\partial F_M}{\partial T_-} = \frac{1}{N}\frac{\partial F_{T_-}}{\partial M} = -\frac{\kappa M^{N-1}}{B_-} \ , \tag{47}$$

$$m_{MT_-}^2 \equiv \frac{\kappa m_Q^*(N-1)M^{N-2}}{\sqrt{2}} \ . \tag{48}$$

Since we are looking for supersymmetry-*breaking* effects, we should carry out our calculations up to the order of the largest non-vanishing contribution in the off-diagonal block, namely $m_{MM} \sim \mathcal{O}(\lambda^4)$. Fortunately, the situation is simplified by noting that this sector includes the Goldstino. Indeed the 3×3 blocks on the diagonal coincide with the fermionic mass matrix and hence vanish. This is sufficient to show that the determinant of this matrix is negative without keeping track of such small orders in λ. We thus get the same instability as we had in section 3.

It is worth noting that any contribution from the CW potential would be proportional to either ξ_{SB} or to ξ_{BT}. On the other hand, the tree-level tachyonic mass is proportional to ξ_{SB}. Since $\xi_{BT} < \xi_{\mathrm{SB}}$, the CW contribution cannot compete with the tree-level contribution.

References

[1] E. Witten, Nucl. Phys. B **188**, 513 (1981).

[2] I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. B **241**, 493 (1984).

[3] I. Affleck, M. Dine and N. Seiberg, Phys. Lett. B **137**, 187 (1984).

[4] Y. Shadmi and Y. Shirman, Rev. Mod. Phys. **72**, 25 (2000) [arXiv:hep-th/9907225].

[5] E. Poppitz and S. P. Trivedi, Ann. Rev. Nucl. Part. Sci. **48**, 307 (1998) [arXiv:hep-th/9803107].

[6] M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, Phys. Rev. D **53**, 2658 (1996) [arXiv:hep-ph/9507378].

[7] M. A. Luty, in *Boulder 2004, Physics in D ≥ 4*, 495-582 [arXiv:hep-th/0509029].

[8] Y. Shadmi, in *Les Houches 2005, Particle physics beyond the standard model*, 147-180 [arXiv:hep-th/0601076].

[9] G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, JHEP **9812**, 027 (1998) [arXiv:hep-ph/9810442].

[10] L. Randall and R. Sundrum, Nucl. Phys. B **557**, 79 (1999) [arXiv:hep-th/9810155].

[11] M. Dine, A. E. Nelson and Y. Shirman, Phys. Rev. D **51**, 1362 (1995) [arXiv:hep-ph/9408384].

[12] See also: G. F. Giudice and R. Rattazzi, Phys. Rept. **322**, 419 (1999) [arXiv:hep-ph/9801271].

[13] I. Dasgupta, B. A. Dobrescu and L. Randall, Nucl. Phys. B **483**, 95 (1997) [arXiv:hep-ph/9607487].

[14] N. Arkani-Hamed, C. D. Carone, L. J. Hall and H. Murayama, Phys. Rev. D **54**, 7032 (1996) [arXiv:hep-ph/9607298].

[15] I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. B **256**, 557 (1985).

[16] E. Poppitz and S. P. Trivedi, Phys. Rev. D **55**, 5508 (1997) [arXiv:hep-ph/9609529].

[17] N. Arkani-Hamed, M. A. Luty and J. Terning, Phys. Rev. D **58**, 015004 (1998) [arXiv:hep-ph/9712389].

[18] N. Arkani-Hamed, J. March-Russell and H. Murayama, Nucl. Phys. B **509**, 3 (1998) [arXiv:hep-ph/9701286].

[19] S. Dimopoulos, G. R. Dvali, R. Rattazzi and G. F. Giudice, Nucl. Phys. B **510**, 12 (1998) [arXiv:hep-ph/9705307].

[20] H. Murayama, Phys. Rev. Lett. **79**, 18 (1997) [arXiv:hep-ph/9705271].

[21] H. Ooguri and Y. Ookouchi, Nucl. Phys. B **755**, 239 (2006) [arXiv:hep-th/0606061].

[22] T. Banks, arXiv:hep-ph/0606313.

[23] M. Dine, J. L. Feng and E. Silverstein, Phys. Rev. D **74**, 095012 (2006) [arXiv:hep-th/0608159].

[24] H. Murayama and Y. Nomura, arXiv:hep-ph/0701231.

[25] M. Dine and J. Mason, arXiv:hep-ph/0611312.

[26] R. Kitano, H. Ooguri and Y. Ookouchi, Phys. Rev. D **75**, 045022 (2007) [arXiv:hep-ph/0612139].

[27] C. Csaki, Y. Shirman and J. Terning, arXiv:hep-ph/0612241.

[28] S. A. Abel and V. V. Khoze, arXiv:hep-ph/0701069.

[29] K. Intriligator, N. Seiberg and D. Shih, JHEP **0604**, 021 (2006) [arXiv:hep-th/0602239].

[30] N. Seiberg, Nucl. Phys. B **435**, 129 (1995) [arXiv:hep-th/9411149].

[31] T. Banks, arXiv:hep-ph/0510159.

[32] K. A. Intriligator and S. D. Thomas, Nucl. Phys. B **473**, 121 (1996) [arXiv:hep-th/9603158].

[33] K. I. Izawa and T. Yanagida, Prog. Theor. Phys. **95**, 829 (1996) [arXiv:hep-th/9602180].

[34] R. Argurio, M. Bertolini, S. Franco and S. Kachru, JHEP **0701**, 083 (2007) [arXiv:hep-th/0610212].

[35] N. Seiberg, Phys. Rev. D **49**, 6857 (1994) [arXiv:hep-th/9402044].

[36] Z. Chacko, M. A. Luty and E. Ponton, JHEP **9812**, 016 (1998) [arXiv:hep-th/9810253].

[37] T. Banks, arXiv:hep-th/0007146.

[38] E. Poppitz and S. P. Trivedi, Phys. Lett. B **401**, 38 (1997) [arXiv:hep-ph/9703246].